A Flexible Approach Combining the Spectral Domain Method and Impedance Boundary Condition for the Analysis of Microstrip Lines

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Abstract—**Impedance boundary condition model is incorporated in the spectral domain formulation to calculate the transmission characteristics of microstrip line with lossy conducting strip. Subsectional rectangular pulse functions are used as the basis functions for the surface current distribution because of the finite conductivity of the conducting strip. The approach has the advantage of being more flexible without presuming the edge condition of the surface current distribution and numerically efficient. Numerical results for the phase and attenuation constants of superconducting microstrip line are computed for a comparison.**

I. Introduction

The spectral domain method has been one of the most popular numerical techniques for the analysis of planar guided-wave structure. It has been applied in the past to a number of different configurations. The two-dimensional version has been applied to the analysis of printed transmission lines including the microstrip line, finline and coplanar waveguide [1]–[5]. The three-dimensional version was applied to finite size structures such as resonators and discontinuities [6], [7]. More recently, the method was extended to the case where the conducting strip is not a perfect conductor [8]. In such a formulation, the strip is treated as and impedance boundary on which the tangential electric field is no longer zero [9].

In most two-dimensional analyses, the basis functions used for expansion of unknown currents or the unknown slot fields for the structures with perfect conducting strips have been the entire-domain functions incorporating the edge conditions [10], [11]. As for the case when the conducting strip is not a perfect conductor, current density at the edge of the strip should remain finite [12], rather than infinite. When solving this kind of lossy structure with the spectral domain method, edge condition at the conducting strip edge need not be presumed if the localized functions are used as the basis functions. The main advantage of using the localized basis functions is the flexibility of the technique. It is not necessary to assume the qualitative nature of the unknown quantity which will be expanded by the basis functions.

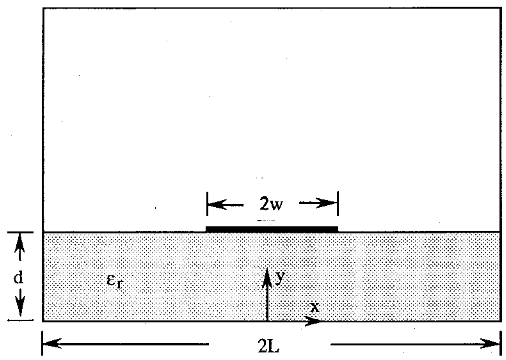

Fig. 1. Enclosed microstrip line with lossy conducting strip.

The extension of the spectral domain approach that incorporates the impedance boundary condition is applied in this paper to analyze the transmission characteristics of microstrip lines with lossy conducting strips. Subsectional rectangular pulse functions are used as the basis functions for the surface current distribution on the conducting strip. The proposed approach in this letter has the advantage of combining the versatility of the spectral domain method in solving the planar guided-wave structure and the flexibility of the localized basis functions in expanding the unknown quantities. Expansion coefficients of the unknown surface current distribution are adjusted automatically in the numerical process to represent the real surface current distribution on the conducting strip.

Numerical results of phase and attenuation constants of a superconducting microstrip line are computed as an example. The results are compared with existing data that were obtained by assuming the edge condition of the surface current distribution at the edge of the superconducting strip.

II. Analysis

Fig. 1 shows the structure of a shielded microstrip line with lossy conducting strip. The field components of the hybrid guided waves are expressed in terms of $\tilde{E}_z(\alpha_n, y)$ and $\tilde{H}_z(\alpha_n, y)$ that are the Fourier transforms in the x-direction of the axial field components $E_z(x, y)$ and $H_z(x, y)$,

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assuming a z-dependence of $e^{-j\beta z}$. The tangential electric and magnetic fields are matched at the dielectric-air interface, $y = d$. The conducting strip is treated as an impedance sheet as part of the boundary condition at the interface. An impedance sheet is characterized by a jump discontinuity in the values of the tangential magnetic fields, but not in the tangential electric fields. These conditions that exist on the surface of the conducting strip are written as

$$\hat{y} \times (E^{+} - E^{-}) = 0 \quad (1a)$$

$$\hat{y} \times (H^{+} - H^{-}) = -\frac{1}{Z}\hat{y} \times (\hat{y} \times E^{+}) = J_s, \quad (1b)$$

where the plus and minus sign superscripts refer to field components above and below the sheet, $\hat{y}$ is the unit normal vector, Z is the uniform surface impedance of the impedance sheet and J_s is surface current density on the sheet.

After some mathematical manipulations, a coupled set of equations relating the Fourier transforms of the unknown surface current densities $\tilde{J}_{sx}(\alpha_n)$ and $\tilde{J}_{sz}(\alpha_n)$ to the Fourier transforms of the tangential electric fields is obtained as

$$[\tilde{Z}_{xx}(\alpha_n) - Z] J_{sx}(\alpha_n) + \tilde{Z}_{xz}(\alpha_n)\tilde{J}_{sz}(\alpha_n) = \tilde{E}_x(\alpha_n), \quad (2a)$$

$$\tilde{Z}_{zx}(\alpha_n) J_{sx}(\alpha_n) + [\tilde{Z}_{zz}(\alpha_n) - Z]\tilde{J}_{sz}(\alpha_n) = \tilde{E}_z(\alpha_n), \quad (2b)$$

where $\tilde{Z}_{xx}$, $\tilde{Z}_{xz}$, $\tilde{Z}_{zx}$ and $\tilde{Z}_{zz}$ are the Fourier-transformed Green's functions of the microstrip line structure in the spectral domain. Since the conducting strip has finite conductivity, the surface current distribution may not be the same as that for the perfect conductor case. The subsectional rectangular pulse functions are applied in this letter as the basis functions for the surface current distribution to handle this situation. Without presuming the edge condition of the surface current distribution, this approach is more flexible. By using the rectangular pulse functions as basis functions, the expansion coefficients are adjusted automatically in the numerical process to represent the real surface current distribution on the conducting strip. The Galerkin's method is applied to (2) to construct a determinant equation for solving the complex propagation constant β.

III. Numerical Results

A computer program was written based on the previous approach. In order to confirm the validity of program, numerical results of a microstrip line with perfect conducting strip were calculated and compared with available data [2]. Difference between our data and those in [2] is indistinguishable. Next, we compare our numerical results with existing data for low T_c superconducting microstrip line. Since most superconducting films are fabricated to be very thin, the width-to-thickness ratio of these films are very large. As a result, the tangential electric to magnetic field ratio on the film surface is almost the same as the measured surface impedance of the superconducting film. The feature makes the impedance boundary condition a good model to use in the spectral domain method to solve for the transmission characteristics of the superconducting microstrip lines.

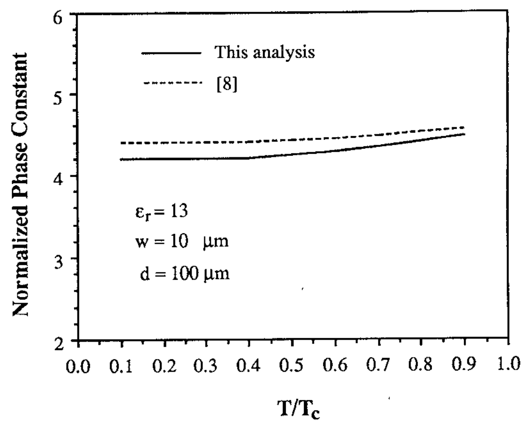

Fig. 2. Comparison of results of a low-T_c NbN superconducting microstrip line. The critical temperature of the NbN film is T_c-12.15°K and the penetration depth at 0°K is $\lambda_0 = 3200$ Å.

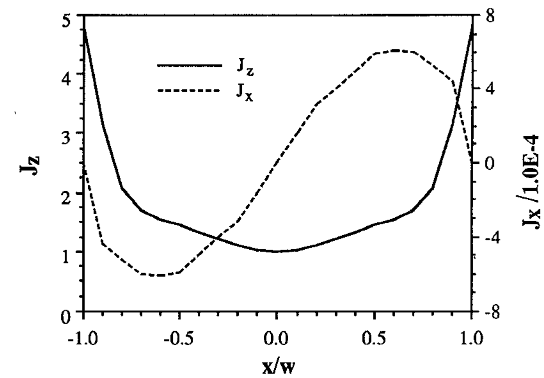

Fig. 3. Surface current distribution of the superconducting microstrip line in Fig. 2 at 5°K.

Fig. 2 shows the variation of the normalized phase constant with respect to temperature at 5 GHz of a low-T_c NbN superconducting microstrip line on a GaAs substrate with $\epsilon_r = 13$. The data in [8] were computed with current basis functions assuming edge condition at the superconducting strip edge while our data were obtained with subsectional rectangular pulse functions. As a consequence, there is some difference between the two results.

Extensive convergence tests were also performed to confirm the accuracy of the numerical results. The normalized phase constant converges monotonically. Typically, it will converge to a steady-state value after $N = 7$ rectangular pulse functions are used for the current basis functions, with the superconducting strip discretized into $2N$ subsections. Fig. 3 shows the surface current distribution on the superconducting strip at 5 GHz. For the axial surface current component J_{sz}, the current density value at the edge of the strip is only about five times the value at the center of the strip. This observation justifies the use of subsectional rectangular pulse functions as the current basis function. Presuming the edge condition for the current basis functions may not be appropriate.

IV. Conclusion

Numerical results based on the proposed approach are computed and compared with published data for the low T_c superconducting microstrip line. This extended spectral domain approach that incorporates the impedance boundary condition is a flexible and efficient method of solving the microstrip line with lossy conducting strip structures by using the subsectional rectangular pulse functions as the current basis functions.

The proposed approach is flexible because the subsectional rectangular pulse functions simulated the true current distribution on the conducting strip better. Numerical results of the surface current distribution justifies this assumption. The spectral domain method is also proven very efficient in solving the microstrip line problem. One other advantage of this approach is the capability of the rectangular pulse functions in handling the structure when the surface impedance of the conducting strip is nonuniform. This topic will be studied in the future work.

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